Indian National Olympiad in Informatics, 2002

Time: 3 hours *1 May, 2002*

Attempt all questions.

1. You are given n boxes, numbered $1, 2, \ldots, n$, and a number p, $1 \leq p \leq n$. You have to interchange the contents of boxes $p, p{+}1, \ldots, n$ with the contents of boxes $1, 2, \ldots, p{-}1$. More precisely, you are supposed to rearrange the contents of the boxes so that the original contents of boxes $p, p{+}1, \ldots, n$ are moved (in sequence) to boxes $1, 2, \ldots, (n{-}p){+}1$, and the original contents of boxes $1, 2, \ldots, p{-}1$ are moved (in sequence) to the boxes $(n{-}p) + 2, (n{-}p) + 3, \ldots, n$.

   Here are two examples.

   10 boxes, p = 7

	1	2	3	4	5	6	7	8	9	10
Before	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
After	a_7	a_8	a_9	a_{10}	a_1	a_2	a_3	a_4	a_5	a_6

   10 boxes, p = 3

	1	2	3	4	5	6	7	8	9	10
Before	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
After	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_1	a_2

   There is an important restriction on how you may rearrange the contents: at each step, the *only* operation available is to interchange (swap) the contents of any two boxes (for instance, you can choose box numbers 4 and 7 and move the contents of box 4 to box 7 and simultaneously move the contents of box 7 to box 4).

   Describe an algorithm to rearrange the contents of the boxes as required, using only this basic operation of swapping the contents of two boxes.

2. (a) Consider the following game involving two players, A and B. There is a stack of k coins. The players move alternately, with Player A moving first. In each move, a player can remove either 1 or 2 coins from the stack. The player who removes the last coin wins the game.

   Depending on the initial number of coins k, either Player A or Player B has a strategy that will guarantee a win. For what values of k will Player A win? For what values of k will Player B win? Describe the winning strategy in both cases.

(b) Consider a variation of the earlier game in which you have two stacks of coins that initially contain m and n coins, respectively. As before, the players move alternately, with Player A moving first. In each move, a player can remove either 1 or 2 coins from one of the two stacks (but if 2 coins are removed, both must be from the same stack). As before, the player who removes the last coin overall wins the game.

Once again, depending on the initial number of coins m and n, either Player A or Player B has a strategy that will guarantee a win. For what values of m and n will Player A win? For what values of m and n will Player B win? Describe the winning strategy in both cases.

3. (a) Our town is the terminus of a railway line. After passengers have disembarked, the carriages are rearranged and the train is prepared for departure.

The train yard has a peculiar arrangement, as shown in Figure 1. Trains arrive from the right. There is a *rearranging yard* that can rearrange 8 carriages at a time, in any order. However, the trains that run are normally 16 carriages long. After the rearranging yard, there are two long sidings that can hold up to 16 carriages each.

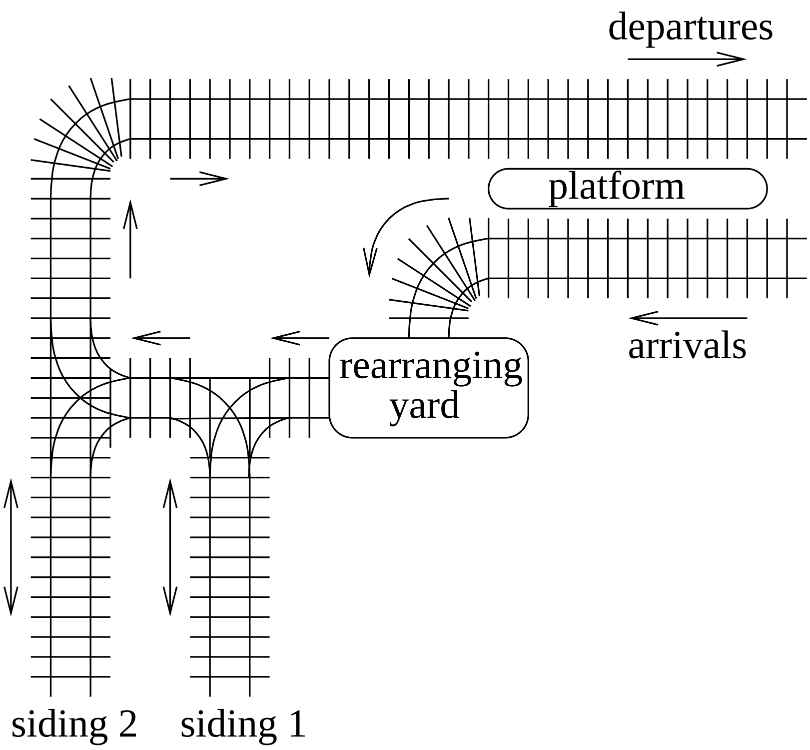

Figure 1:

Carriages can be added and removed one at a time from the two sidings, but carriages cannot move backwards from the second siding to the first, from the first siding back to the rearranging yard or from the rearranging yard back to the platform. When the train arrives at the departure platform the carriages must be in the correct order.

Describe an algorithm for rearranging trains with 16 carriages in any order that one wants using this shunting arrangement. Note that the rearrangement may involve mixing up carriages from the first half of the train and the second half. For instance, the incoming train could be

$$c_1c_2c_3c_4c_5c_6c_7c_8c_9c_{10}c_{11}c_{12}c_{13}c_{14}c_{15}c_{16}$$

and the outgoing train could be required to be

$$c_1c_3c_5c_7c_9c_{11}c_{13}c_{15}c_2c_4c_6c_8c_{10}c_{12}c_{14}c_{16}.$$

(b) When upgrading the station, it was noticed that the rearranging yard could be dispensed with and replaced by additional sidings. Show that for any value of n, the arrangement of n sidings in Figure 2, where carriages can move from one siding to the next but not backwards, can be used to rearrange a train with 2^n carriages in any desired order. Assume that each siding is long enough to hold all 2^n carriages.

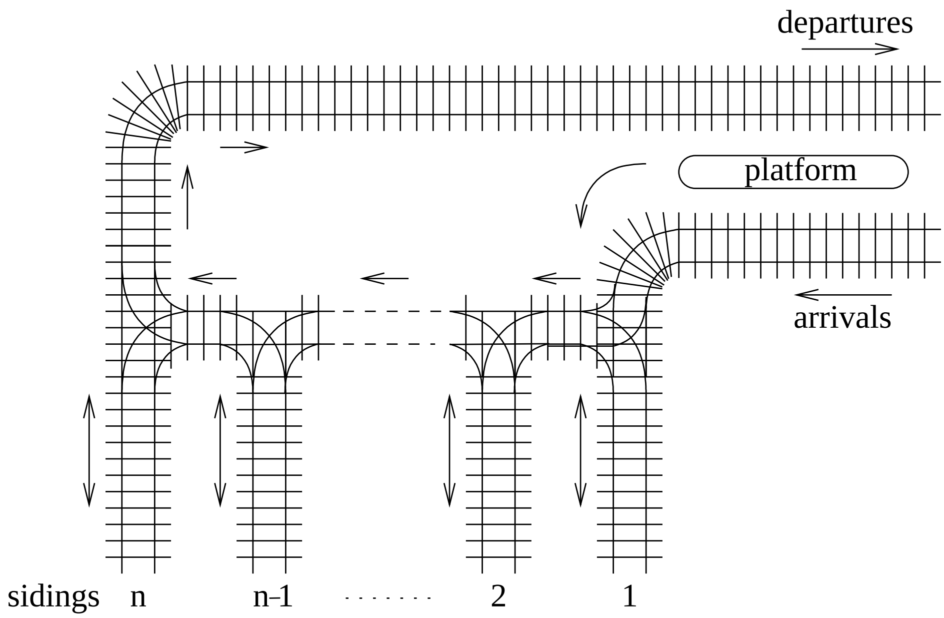

Figure 2: