Zonal Informatics Olympiad, 2002–2003

Solutions

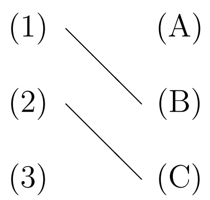
Explanations for the solutions are given separately, starting on the next page.

1. (a) Grouping: (12,108), (24, 103), (25,83), (65,77), (66, 69)
   Weight: 142

   (b) Grouping: (-23,247), (-16,161) (72,152) (75,112), (77,83)
   Weight: 224

   (c) Grouping: (2,88), (19,81), (28,76), (41,69), (59,61)
   Weight: 120

2. A number is good precisely when it is a multiple of 4.

3. 

1 is compatible with B
2 is compatible with C
3 and A are not compatible with any sequence.

4. (a) Sack contents: 10 kg of B, 10 kg of C
   Total Value: 2833.33

   (b) Sack contents: 10 kg of B, 9 kg of D, 11 kg divided in any way between A and C.
   Total Value: 3920

   (c) Sack contents: 10 kg of B, 15 kg of C, 5 kg of D
   Total Value: 3075

5. (a) Komal: 116

   (b) Narain: 119

   (c) Robert: 97

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Solutions, with explanations

1. (a) Grouping: $\{(12, 108), (24, 103), (25, 83), (65, 77), (66, 69)\}$
   Weight: 142

   (b) Grouping: $\{(-23, 247), (-16, 161)(72, 152)(75, 112), (77, 83)\}$
   Weight: 224

   (c) Grouping: $\{(2, 88), (19, 81), (28, 76), (41, 69), (59, 61)\}$
   Weight: 120

   Justification:

   Suppose we have pairs (a, b) and (c, d) such that $a \leq c$ and $b \leq d$. The weight of these pairs is $\max\{(a+b), (c+d)\} = c+d$. If we swap b and d and make two new pairs (a, d) and (c, b), we get a pairing whose weight is $\max\{(a+d), (c+b)\}$. Since $a \leq c$, $a+d \leq c+d$ and since $b \leq d$, $c+b \leq c+d$. Thus, the new pairing has a smaller weight than the original pairing. It follows that the minimum weight pairing of our list of m numbers will never contain pairs of the form (a, b) and (c, d) with $a \leq c$ and $b \leq d$.

   An efficient way to generate a minimum weight pairing is as follows: Pair up the smallest and largest numbers in the list. Eliminate this pair and once again pair up the smallest and largest numbers from the remainder of the list, etc.

   (a) 103, 24, 77, 65, 12, 108, 69, 25, 66, 83

Best grouping:	(12,108)	(24, 103)	(25,83)	(65,77)	(66, 69)
Weights	120	127	108	142	135

   Weight: $142 = \max\{120, 127, 108, 142, 135\}$.

   (b) 83, 112, -16, 72, 161, 75, 152, -23, 77, 247

Best grouping:	(-23,247)	(-16,161)	(72,152)	(75,112)	(77,83)
Weights	224	145	224	187	160

   Weight: $224 = \max\{224, 145, 224, 187, 160\}$.

   (c) 19, 81, 2, 41, 61, 59, 28, 69, 76, 88

Best grouping:	(2,88)	(19,81)	(28,76)	(41,69)	(59,61)
Weights	90	100	104	110	120

   Weight: $120 = \max\{90, 100, 104, 110, 120\}$.

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2. A number is good precisely when it is a multiple of 4.

   Justification:

   Clearly, no odd number can be good, because the rules generate only even numbers.

   Suppose we start with an even number. After we double it for the first time, we have a multiple of 4. After this, we either double it again (which still leaves us with a multiple of 4) or we subtract 100 (since 100 is a multiple of 4, subtracting 100 leaves us again with a multiple of 4). In other words, not only do we generate only even numbers, we generate only multiples of 4. So, if we start with an even number that is not a multiple of 4, we will never get back to the starting number.

Thus, for a number to be good it must be a multiple of 4.

We also have to prove that every multiple of 4 is indeed good. However, there are only 22 two digit multiples of 4—namely 12,16,…,96—so we can exhaustively verify that each of these is in fact good. These numbers break up into two generating cycles, as follows:

$$
\begin{tikzcd}
12 \arrow[r] & 24 \arrow[r] & 48 \arrow[r] & 96 \arrow[r] & 92 \arrow[r] & 84 \arrow[r] & 68 \arrow[r] & 36 \arrow[r] & 72 \arrow[r] & 44 \arrow[d] \\
56 \arrow[u] & 28 \arrow[l] & 64 \arrow[l] & 32 \arrow[l] & 16 \arrow[l] & 8 \arrow[l] & 4 \arrow[l] & 52 \arrow[l] & 76 \arrow[l] & 88 \arrow[l]
\end{tikzcd}
\qquad
\begin{tikzcd}
20 \arrow[r] & 40 \arrow[d] \\
60 \arrow[u] & 80 \arrow[l]
\end{tikzcd}
$$

3.

$$
\begin{tikzcd}
(1) \arrow[dr, dash] & (A) \\
(2) \arrow[dr, dash] & (B) \\
(3) & (C)
\end{tikzcd}
$$

1 is compatible with B
2 is compatible with C
3 and A are not compatible with any sequence.

Justification:

Input sequences

(1) 5,8,10,3,2,9,7,6,4,1

(2) 10,1,9,6,5,4,2,8,3,7

(3) 7,6,2,5,3,4,8,10,4,9

Output sequences

(A) 6,2,7,4,5,8,3,10,9,4

(B) 3,9,7,2,10,6,1,4,8,5

(C) 9,5,6,1,2,4,10,7,3,8

Since only 3 and A have two 4's and no 1, we can break up the problem into checking the compatibility of 1 and 2 against B and C and 3 against A.

Consider an input sequence 1,2,3. The only output sequence that is *not* compatible with 1,2,3 is 3,1,2. More generally, if the input sequence contains 3 numbers $\ldots a \ldots b \ldots c \ldots$ where $\ldots$ stands for an arbitrary sequence of other numbers in between, and the output sequence contains $\ldots c \ldots a \ldots b \ldots$, then the output is not compatible with the input. Otherwise it is compatible.

To check whether 1 is compatible with B and C, use numbers from 11 to 20 to systematically renumber the sequence in 1 in ascending order and use the same renumbering for B and C. We renumber 1 as

1: 11(5), 12(8), 13(10), 14(3), 15(2), 16(9), 17(7), 18(6), 19(4), 20(1)

Then we have

B: 14(3), 16(9), 17(7), 15(2), 13(10), 18(6), 20(1), 19(4), 12(8), 11(5)

C: 16(9), 11(5), 18(6), 20(1), 15(2), 19(4), 13(10), 17(7), 14(3), 12(8)

Now we see if there are three numbers $11 \leq a < b < c \leq 20$ such that the numbers appear in the order c, a, b in the renumbered version of B or C.

In B, we can check that for each number c, all numbers smaller than c that appear after c appear in descending order — in other words, if we have $a < b < c$ and a, b appear after c in B, then a appears after b, so the three numbers appear as c, b, a, not c, a, b. Thus, there is no order violation in B and B is compatible with 1.

This is not the case with C. For instance, we have the sequence 16,11,15, which corresponds to illegally reordering 5,2,9 from sequence 1 as 9,5,2 in sequence C. So, C is not compatible with 1.

Similarly, we can check sequence 2 against B and C.

2: 11(10), 12(1), 13(9), 14(6), 15(5), 16(4), 17(2), 18(8), 19(3), 20(7)

B: 19(3), 13(9), 20(7), 17(2), 11(10), 14(6), 12(1), 16(4), 18(8), 15(5)

C: 13(9), 15(5), 14(6), 12(1), 17(2), 16(4), 11(10), 20(7), 19(3), 18(8)

In B, we have, for instance, the sequence 19,13,17 which corresponds to illegally reordering 9,2,3 from sequence 2 as 3,9,2 in sequence B. So, B is not compatible with 2.

In C, we can check that for each number c, all numbers smaller than c that appear after c appear in descending order so there is no order violation and C is compatible with 2.

Finally, we use the same technique to renumber and compare 3 and A.

3: 11(7), 12(6), 13(2), 14(5), 15(3), 16(4), 17(8), 18(10), 19(4), 20(9)

A: 12(6), 13(2), 11(7), 16(4), 14(5), 17(8), 15(3), 18(10), 20(9), 19(4)

A contains the illegal sequence 16,14,15 corresponding to reordering 5,3,4 from 3 as 4,5,3 so A is not compatible with 3. (Actually, we should also check the renumbering of A where the first 4 in A is renumbered 19 and the second 4 is renumbered 16. In this case, we have the illegal sequence 19,14,15, so the sequences remain incompatible).

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4. (a) Sack contents: 10 kg of B, 10 kg of C
   Total Value: 2833.33

   (b) Sack contents: 10 kg of B, 9 kg of D, 11 kg divided in any way between A and C.
   Total Value: 3920

   (c) Sack contents: 10 kg of B, 15 kg of C, 5 kg of D
   Total Value: 3075

Justification:

The thief should choose items according to their value per unit weight. Thus, we compute the value per unit weight of each item and fill the sack starting with the highest value item.

(a) $W = 20$

	A	B	C
amount	15	10	18
value	1800	1500	2400
unit value	120	150	133.33

Therefore, pick items in the order B, C, A. After exhausting all 10 kg of B, add 10 kg of C to fill the sack. Total value is $10*150+10*133.33 = 1500+1333.33 = 2833.33$.

(b) $W = 30$

	A	B	C	D
amount	25	10	15	9
value	3000	1400	1800	1200
unit value	120	140	120	133.33

Therefore, pick items in the order B, D, and then A or C. After exhausting all 10 kg of B and all 9 kg of D, add 11 kg of A/C to fill the sack. Total value is $10 * 140 + 9 * 133.33 + 11 * 120 = 1400 + 1200 + 1320 = 3920$.

(c) $W = 30$

	A	B	C	D
amount	25	10	15	20
value	2250	1100	1500	1900
unit value	90	110	100	95

Therefore, pick items in the order B, C, D, A. After exhausting all 10 kg of B and all 15 kg of C, add 5 kg of D to fill the sack. Total value is $10*110+15*100+5*95 = 1100 + 1500 + 475 = 3075$.

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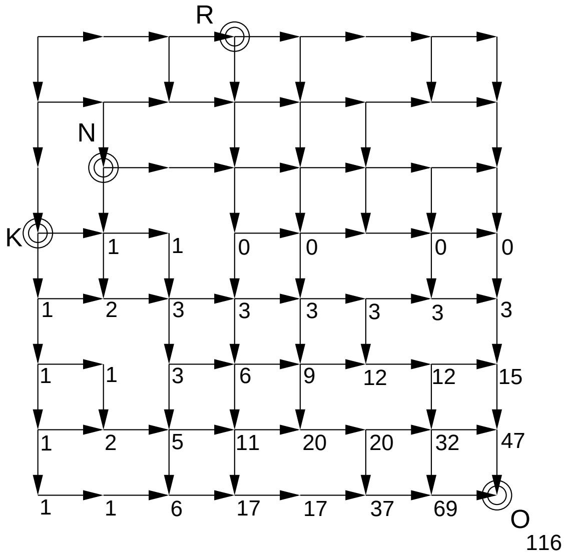
5. (a) Komal: 116

   (b) Narain: 119

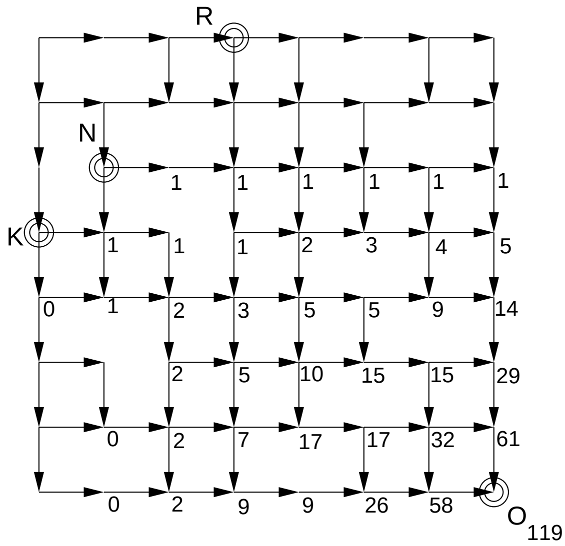
   (c) Robert: 97

Justification:

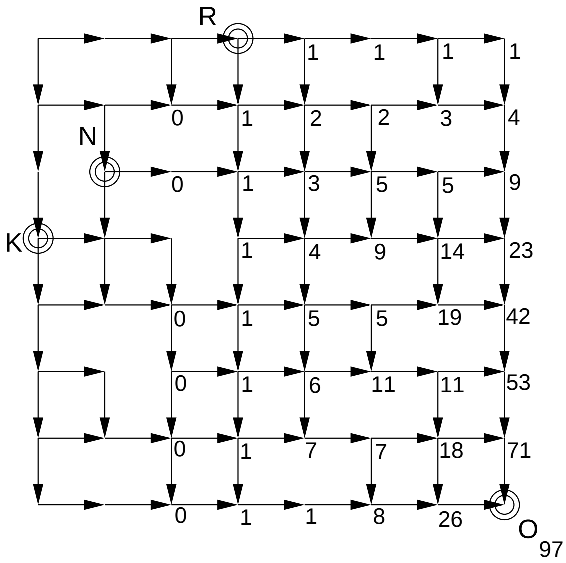
To count the number of ways to reach an intersection, we can add up the number of ways of reaching each intersection that is one step back from the current intersection. Since all the roads in the map are one way pointing down or to the right, each intersection has at most two immediately preceding intersections, above and to the left. We can thus start from the intersections marked K, N and R and systematically fill up row by row the number of ways of reaching each intersection, until we arrive at the figure we want, for the intersection marked O. The computation for each of the three cases is shown below.



Komal



Narain



Robert