# Zonal Informatics Olympiad, 2012

*Instructions to candidates*

1. The duration of the examination is 3 hours.
2. Calculators, log tables and other aids are not permitted.
3. The question paper carries 80 marks, broken up into four questions of 20 marks each. Each question has three parts. *If you solve all three parts correctly, you get 20 marks for that question.* Otherwise, you get 5 marks for each part that you solve correctly.
4. Attempt all questions. There are no optional questions.
5. There is a separate *Answer Sheet*. To get full credit, you *must* write the final answer in the space provided on the Answer Sheet.
6. Write *only* your final answers on the Answer Sheet. Do *not* use the Answer Sheet for rough work. Submit all rough work on separate sheets.
7. Make sure you fill out your contact details on the Answer Sheet as completely and accurately as possible. We will use this information to contact you in case you qualify for the second round.

# Zonal Informatics Olympiad, 2012

*Questions*

1. You have a bag with some black balls and some white balls. You put your hand into the bag and pull out a pair of balls.

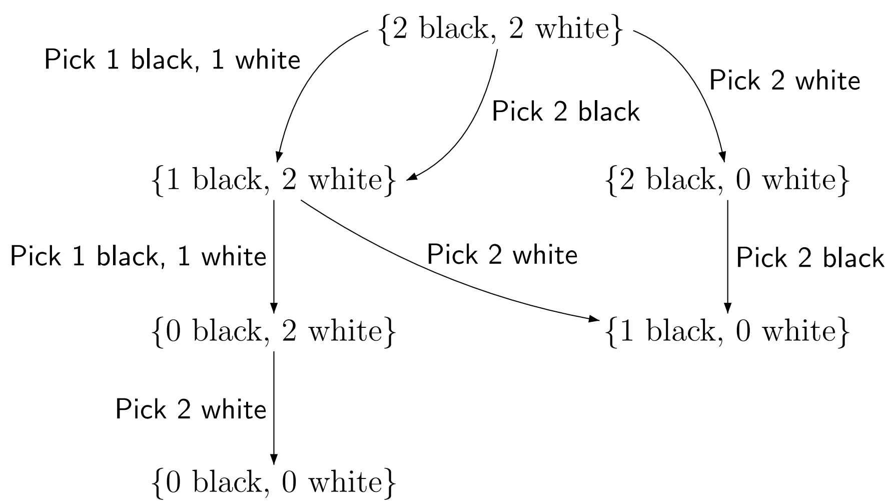
   (a) If both balls are black, you throw away one and return one to the bag.

   (b) If both balls are white, you throw them both away.

   (c) If the balls are of different colours, you throw away the black ball and return the white ball to the bag.

   You can keep doing this so long as there are at least two balls left in the bag. The process stops when the bag is empty or has only a single ball left.

   For instance, suppose we start with 2 black balls and 2 white balls. The various ways the game can evolve are described below.




   Suppose you start with a given number of black and white balls, play the game *and you end up with a single ball in the bag.* Let $N$ denote the number of times you pulled out pairs of balls. We are interested in answering the following questions.

   - What are the minimum and maximum values for $N$ for which you end with a single ball in the bag?
   - Is the last ball remaining in the bag always white, always black or of either colour?

In the example worked out above, if we end up with a single ball in the bag, it must be black. Also, the minimum and maximum values of $N$ are both 2. Note that we are not interested in the value of $N$ in situations where the bag becomes empty.

Compute the minimum and maximum values of $N$ as well as the possibilities for the colour of the last ball for each of the following initial contents of the bag.

(a) 17 black balls, 23 white balls
(b) 42 black balls, 32 white balls
(c) 111 black balls, 99 white balls

2. A toy set contains blocks showing the numbers from 1 to 9. There are plenty of blocks showing each number and blocks showing the same number are indistinguishable. We want to examine the number of different ways of arranging the blocks in a sequence so that the displayed numbers add up to a fixed sum.

   For example, suppose the sum is 4. There are 8 different arrangements:

   1 1 1 1
   1 1 2
   1 2 1
   1 3
   2 1 1
   2 2
   3 1
   4

   The rows are arranged in dictionary order (that is, as they would appear if they were listed in dictionary).

   In each of the cases below, you are given the desired sum $S$ and a number $K$. You have to write down the $K^{th}$ line when all arrangements that add up to $S$ are written down as described above. For instance, if $S$ is 4 and $K$ is 5, the answer is **2 1 1**. Remember that $S$ may be large, but the numbers on the blocks are only from 1 to 9.

   (a) $S = 9$, $K = 156$
   (b) $S = 11$, $K = 881$
   (c) $S = 14$, $K = 4583$

3. You are given a grid of cells. Each cell has a positive integer written on it. You can *move* from a cell $x$ to a cell $y$ if $x$ and $y$ are in the same row or same column, and the number in $y$ is strictly smaller than the number in $x$. You want to colour some cells red so that:

   - Every cell can be reached by starting at a red cell and following a sequence of zero or more moves as defined above.
   - The number of red cells is as small as possible.

   You should report the following information:

- The number of red cells.
- The smallest number appearing amongst all red cells.

If there are multiple valid solutions, give any one solution.

For example, suppose the grid is as follows:

| | | |
|---|---|---|
| 2 | 2 | 3 |
| 2 | 1 | 2 |
| 3 | 2 | 2 |

It is sufficient to colour the two cells labelled 3, and one cannot do better than this. In this case, the number of red cells is 2 and the smallest number appearing amongst all red cells is 3.

(a)

| | | | | | | |
|---|---|---|---|---|---|---|
| 55 | 25 | 49 | 40 | 55 | 3 | 55 |
| 33 | 32 | 26 | 59 | 41 | 40 | 55 |
| 31 | 23 | 41 | 58 | 59 | 14 | 33 |
| 9 | 19 | 9 | 40 | 4 | 40 | 40 |
| 55 | 54 | 55 | 46 | 52 | 39 | 41 |
| 10 | 41 | 7 | 47 | 5 | 30 | 54 |
| 40 | 22 | 31 | 36 | 7 | 40 | 28 |
| 21 | 40 | 41 | 59 | 14 | 36 | 31 |

(b)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 50 | 98 | 54 | 6 | 34 | 94 | 63 | 52 | 39 |
| 62 | 46 | 75 | 28 | 65 | 18 | 37 | 18 | 97 |
| 13 | 80 | 33 | 69 | 93 | 78 | 19 | 40 | 13 |
| 94 | 10 | 88 | 43 | 61 | 72 | 94 | 94 | 94 |
| 41 | 79 | 82 | 27 | 71 | 62 | 57 | 67 | 34 |
| 8 | 93 | 2 | 12 | 93 | 52 | 91 | 86 | 93 |
| 94 | 79 | 64 | 43 | 32 | 94 | 42 | 91 | 9 |
| 25 | 73 | 29 | 31 | 19 | 70 | 58 | 12 | 11 |

(c)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 54 | 6 | 34 | 78 | 63 | 52 | 39 | 41 | 46 |
| 75 | 28 | 65 | 18 | 37 | 18 | 13 | 80 | 33 | 69 |
| 78 | 19 | 40 | 13 | 10 | 43 | 61 | 72 | 13 | 46 |
| 56 | 41 | 79 | 82 | 27 | 71 | 62 | 57 | 67 | 81 |
| 8 | 71 | 2 | 12 | 52 | 81 | 1 | 79 | 64 | 81 |
| 32 | 41 | 9 | 25 | 73 | 29 | 31 | 19 | 41 | 58 |
| 12 | 11 | 41 | 66 | 63 | 14 | 39 | 71 | 38 | 16 |
| 71 | 43 | 70 | 27 | 78 | 71 | 76 | 37 | 57 | 12 |
| 77 | 50 | 41 | 81 | 31 | 38 | 24 | 25 | 24 | 81 |

4. A shop gets its supplies in cartons, each with a unique serial number. These cartons are stored in a cupboard with many shelves. The shelves are arranged from top to bottom. In each shelf, cartons are arranged from left to right.

   The shop assistant uses a strange system to store cartons on the shelves. When a carton numbered $K$ arrives in the shop, he places it according to the following rules.

   (a) If $K$ is bigger than the serial numbers of all the cartons on the top shelf, he adds $K$ at the right end of the top shelf.

(b) Otherwise, he finds the carton with the smallest serial number bigger than $K$ in the top shelf. Suppose this carton has number $L$. He replaces carton $L$ by carton $K$ and inserts carton $L$ into the second shelf using the same pair of rules.

In other words, if $L$ is bigger than all the serial numbers in the second shelf, carton $L$ is added at the right end of the second shelf.

Otherwise, it displaces carton $M$, where $M$ is the smallest serial number on the second shelf that is bigger than $L$. Carton $M$ is then inserted into the third self using the same pair of rules, and so on.

There are always enough empty shelves with adequate free space on each shelf to complete this process.

Separately, he records in his notebook the order in which positions get filled in the shelves.

Here is an example to illustrate the method. Suppose six cartons numbered 32, 43, 21, 65, 54 and 11 arrive in that order. Then, the arrangement of the cartons on the shelves and the record in the notebook after each item arrives is as follows.

| *Event* | *Shelf contents* | *Notebook* |
|---|---|---|
| 32 arrives | 32 | 1 |
| 43 arrives | 32 43 | 1 2 |
| 21 arrives,<br>bumps 32 to second shelf | 21 43<br>32 | 1 2<br>3 |
| 65 arrives | 21 43 65<br>32 | 1 2 4<br>3 |
| 54 arrives,<br>bumps 65 to second shelf | 21 43 54<br>32 65 | 1 2 4<br>3 5 |
| 11 arrives,<br>bumps 21 to second shelf,<br>which bumps 32 to third shelf | 11 43 54<br>21 65<br>32 | 1 2 4<br>3 5<br>6 |

Remember that the notebook records the order in which the positions on the shelves are occupied. Thus, the entry 5 in the final notebook after all six items have arrived indicates that the second position in the second shelf became occupied when the fifth carton arrived, even though the actual carton in that position, 65, was the fourth carton to arrive, not the fifth.

The manager of the shop has a problem. From the arrangement of cartons on the shelves and the record in the notebook, he must figure out the order in which the cartons originally arrived, so that old stocks don't accumulate. For instance, from the final pair of entries in the table above, he should recover the fact that the original sequence of cartons was 32, 43, 21, 65, 54, 11.

Each case below describes the contents of the shelves and the corresponding entries in the notebook after a sequence of cartons has arrived. You have to calculate and write down the original sequence in which the cartons arrived.

| | *Final Shelf contents* | *Final Notebook entry* |
|---|---|---|
| (a) | 8 11 86<br>12 95<br>22<br>47 | 1 3 5<br>2 7<br>4<br>6 |
| (b) | 6 34 52 66<br>39 63<br>50 98<br>54 | 1 4 5 6<br>2 8<br>3 9<br>7 |
| (c) | 8 22 47 78<br>11 33 86<br>12 95<br>28<br>40 | 1 3 6 7<br>2 4 8<br>5 11<br>9<br>10 |

# Zonal Informatics Olympiad, 2012: *Answer sheet*

| **Name:** | **Class:** | **Sex:** |
|---|---|---|
| **School:** | | |
| **Examination Centre:** | | |
| **Father or Mother's Name:** | | |
| **Full home address with PIN code:** | | |
| **Home phone number, with STD Code:** | | |
| **Email address:** | | |

*Write only your final answers in the space provided. Write all rough work on separate sheets.*

| ***Question 1*** | | | |
|---|---|---|---|
| | Min $N$ | Max $N$ | Colour of last ball (Black/White/Either) |
| (a) | | | |
| (b) | | | |
| (c) | | | |

| ***Question 2*** | |
|---|---|
| | $K^{th}$ row of arrangements adding up to $S$ |
| (a) | |
| (b) | |
| (c) | |

| ***Question 3*** | | |
|---|---|---|
| | Number of red cells | Smallest number among red cells |
| (a) | | |
| (b) | | |
| (c) | | |

| ***Question 4*** | |
|---|---|
| | Original sequence in which cartons arrived at the shop |
| (a) | |
| (b) | |
| (c) | |

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| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. | a | b | c | | 2. | a | b | c | | |
| 3. | a | b | c | | 4. | a | b | c | | **Total** |