Zonal Informatics Olympiad, 2016

Instructions to candidates

1. You might find a separate window called "Debugger Control", which has two buttons "Start Debugger" and "Show Debugger". If you do, **do not** click on those buttons in that window. You can move the window to the side and continue your work. Clicing on any of the two buttons in that window will get you **disqualified**.

2. The question paper carries 80 marks, broken up into four problems of 20 marks each. Each problem has three parts. If you solve all three parts correctly, you get 20 marks for that question. Otherwise, you get 5 marks for each part that you solve correctly.

3. All the 4*3 = 12 parts appear as separate Questions in the right panel ("Question Palette").

   The first three Questions correspond to the three parts of Problem 1, the next three correspond to the three parts of Problem 2 and so on.

4. Attempt all questions. There are no optional questions.

5. There are no negative marks.

6. All expected answers are integers. Type in only the integer. So, if your answer is "162", enter only "162". Not "0162", or "162.0", etc.

7. Remember to save each answer. Only your final saved answers will be considered.

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Questions

1. A binary string of length N is a sequence of $0s$ and $1s$. For example, 01010 is a binary string of length 5. For a string A we write A_i to refer to the letter at the i^{th} position. Positions are numbered starting from 1. For example, if $A = 01010$ then $A_1 = 0$ and $A_4 = 1$.

   Let A be a string of length N. We say that a string B is a substring of A if $B = A_i A_{i+1} ... A_j$ for some $1 \leq i \leq j \leq N$. So, 1 is a substring of $A = 01010$, since $1 = A_2$. The string 101 is a substring of A because $101 = A_2 A_3 A_4$.

   In this problem you have to count the number of binary strings of length N which contain 11011 as a substring. Take, for example, $N = 6$. We have 4 binary sequences of length 6 which contain 11011 as a substring. They are $011011, 111011, 110110$ and 110111. Therefore, the answer for $N = 6$ is 4.

   Your task is to report the answer for three values of N.

   (a) $N = 9$: How many binary strings of length 9 contain 11011 as a substring?

   (b) $N = 10$: How many binary strings of length 10 contain 11011 as a substring?

   (c) $N = 11$: How many binary strings of length 11 contain 11011 as a substring?

2. Uncle Shiva has gifted Nikhil a new toy. Uncle Shiva believes in gifting toys with a mathematical flavour and this is no different. This toy consists of a wooden board with wooden pegs. The pegs are arranged in N columns with 3 pegs in each column. Each peg is coloured $Red, Blue$ or $Green$.

   Nikhil has a set of $N - 1$ elastic bands (i.e. rubber bands). He builds a chain of bands linking column 1 to column N as follows. He starts by placing a band from a peg in column 1 to a peg in column 2. From the peg where the first band ends, he places a second band connecting that peg in column 2 to a peg in column 3. Continuing in this way, he places all $N - 1$ bands to connect the N columns.

   While building this chain of elastic bands, he is not allowed to connect two pegs at the same position in adjacent columns. So for instance, the second peg on column i cannot be connected to the second peg on column $i - 1$ or the second peg on column $i + 1$. It may be connected to any other peg in those two neighbouring columns.

   Uncle Shiva has added a constraint. Nikhil has to arrange the bands so that while moving from column 1 to column N along the bands, the number of red pegs encountered should be even (i.e. 0 or 2 or 4 or $\ldots$).

   Nikhil is way too smart for Uncle Shiva and he can solve this game in a jiffy. Instead, he decides to count the number of ways in which he can solve the game.

   Suppose we have $N = 3$ and the colours of the pegs in the 3 columns are

Column	1	2	3
Peg 1	R	B	B
Peg 2	G	G	G
Peg 3	R	R	B

That is, the first column has 3 pegs coloured *Red*, *Green* and *Red* respectively. The second column has 3 pegs coloured *Blue*, *Green* and *Red* respectively and the last one has 3 pegs coloured *Blue*, *Green* and *Blue* respectively.

One solution is to connect the first peg on column 1 with the third peg on column 2 and then connect the third peg on column 2 with the first peg on column 3. Writing P_j to refer to j^{th} peg in a column, we may represent such a solution as: P_1—P_3—P_1.

The collection of all solutions is enumerated below:

- P_1—P_3—P_1
- P_1—P_3—P_2
- P_2—P_1—P_2
- P_2—P_1—P_3

So, the answer for this arrangment is 4. Your task is to compute the number of solutions for 3 arrangements of pegs.

(a)

Column	1	2	3	4	5	6
Peg 1	R	R	R	R	G	B
Peg 2	R	G	R	B	B	G
Peg 3	G	G	G	R	R	G

(b)

Column	1	2	3	4	5	6
Peg 1	G	G	R	G	B	G
Peg 2	G	R	R	B	G	R
Peg 3	B	B	G	G	R	R

(c)

Column	1	2	3	4	5	6	7
Peg 1	R	G	R	B	B	R	B
Peg 2	G	R	R	G	R	G	R
Peg 3	G	G	G	R	G	R	R

3. The King of Zyorg has a cabinet with N ministers and they have assembled for a meeting. The King enters the meeting chamber to find that the ministers have arranged themselves so that friends sit with friends.

   The seats are numbered $1 \dots N$ and we refer to the minister sitting on the i^{th} seat in this initial arrangment as i. So, the initial arrangment is of the ministers is $(1, 2, 3 \dots N)$.

   The King orders the ministers to rearrange themselves and finds that some of the friends have just exchanged seats. The enraged King then orders that they must rearrange themselves so that for every $k < N$ the set of ministers sitting in the first k chairs **DOES NOT** consist only of the ministers $1, 2, \dots k$.

   He then wonders if this is possible at all and quickly convinces himself that this is so. Then he wonders how many different ways can they rearrange themselves fulfilling his order.

   For example, suppose $N = 3$. The initial arrangment is of course $(1, 2, 3)$. If they rearrange themselves as $(2, 1, 3)$ this will violate the King's order because for $k = 2$ the ministers in positions 1 upto 2 are $\{1, 2\}$. On the other hand the arrangement $(3, 1, 2)$ fulfils the requirement. For $k = 1$ we have $\{3\}$, for $k = 2$ we have $\{3, 1\}$. You can check that there are exactly 3 arrangments that comply with the King's order. They are : $(2, 3, 1), (3, 1, 2)$ and $(3, 2, 1)$. So, for $N = 3$ the answer to this problem is 3.

   Your task is to write out the answers for 3 values of N.

   (a) $N = 5$
   (b) $N = 6$
   (c) $N = 7$

4. Uncle Shiva has given Nikhil yet another toy. It consists of a bag containing a collection of wooden tiles. Each tile has a number written on it and no two tiles in the bag have the same number written on them.

   Uncle Shiva will then call out a number K. Nikhil should pick out a subset of the tiles from the bag so that if tile numbered i and j are in his subset then $i - j$ is not K. That is, for any two tiles in his bag the difference between the numbers written on them should **NOT** be K.

   Of course, Nikhil could just pick the empty subset or say a subset with only one tile and claim to have solved the game. Uncle Shiva expects him to pick a subset with maximum size to win the game.

   Suppose the bag contains tiles with the numbers $1, 2, 3, 4, 5, 9$ and $K = 3$. Then Nikhil can pick $\{1, 2, 3, 9\}$. But he cannot pick $\{1, 2, 4, 9\}$ because 1 and 4 differ by 3. (There are many subsets of size 4 which satisfy the requirement: $\{1, 2, 3, 9\}$, $\{1, 3, 5, 9\}$, $\{2, 3, 4, 9\}$, $\{2, 3, 5, 9\}$). You can check that there is no subset of size more than 4 satisfying the requirement.

Your task is only to report the size of the largest set such that no pair differs by K. For this instance, the answer is 4.

Report the answer for 3 sets of tiles and K:

(a) $S = \{2, 3, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 38\}$
$K = 6$

(b) $S = \{2, 5, 9, 11, 12, 16, 18, 19, 23, 25, 26, 30, 32, 33, 37, 40, 44, 51, 58\}$
$K = 7$

(c) $S = \{2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 25, 26, 29\}$
$K = 7$
