Zonal Informatics Olympiad, 2019

Instructions to candidates

1. The question paper carries 80 marks, broken up into four problems of 20 marks each. Each problem has three Test Cases. If you solve all three Test Cases correctly, you get 20 marks for that problem. Otherwise, you get 5 marks for each Test Case that you solve correctly.

2. All the $4 \times 3 = 12$ Test Cases appear as separate Questions in the right panel ("Question Palette").

   The first three Questions correspond to the three Test Cases of Problem 1, the next three correspond to the three Test Cases of Problem 2 and so on.

   A question icon turning green in the Question Palette, does not mean that it is correct. It just denotes that you have attempted it. All the questions will be evaluated later.

3. Attempt all questions. There are no optional questions.

4. There are no negative marks.

5. All expected answers are integers. Type in only the integer. So, if your answer is "162", enter only "162". Not "0162", or "162.0", etc.

6. Remember to save each answer. Only your final saved answers will be considered.

7. Near the top-right corner, you should be able to see a calculator icon. Clicking it pops up a calculator which you may use.

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Questions

1. A sequence of positive integers, $a[1], a[2], a[3], \ldots, a[n]$ is called a *Special Sequence*, if $a[1]$ divides $a[2]$, $a[2]$ divides $a[3]$, and so on until $a[n-1]$ divides $a[n]$, and if all the elements are distinct. For example, $(2, 4, 8, 32)$ is a Special Sequence. But $(4, 2, 8)$ is not, because 4 does not divide 2. Similarly $(2, 4, 4, 8)$ is also not Special, because the elements are not distinct.

   You need to find the number of Special Sequences such that all the elements of the sequence are from the set $\{1, 2, \ldots, K\}$.

   Suppose $K = 3$. The Special Sequences possible are $(1), (2), (3), (1, 2), (1, 3)$. So the answer would be 5.

   Find the answer for the following values of K:

   (a) $K = 15$

   (b) $K = 19$

   (c) $K = 22$

2. You have developed an artificial amoeba, and you can control exactly how it divides. Each individual amoeba can be instructed to divide into A, B, or C amoebas. That is, if you instruct an amoeba to divide into A, this amoeba will disappear, and A different new amoeba will appear.

   You start out with K amoeba initially, and you want to give them instructions such that at the end, you have exactly N amoeba left. Giving an instruction is a costly affair because it requires you to produce some biochemicals, and so you want to give as few instructions as possible. Find and write the minimum number of instructions that you should give to end up with exactly N amoebas. If it cannot be done, write -1 instead. Note that each instruction is given to a single amoeba, and not all of them together.

   For example, suppose $K = 1, A = 1, B = 2, C = 3, N = 4$. Then, you can take the single amoeba, instruct it to divide into $B(2)$ amoebas. Now, there are 2 amoebas. Then take one of these amoebas and instruct it to divide into $C(3)$ amoebas. So now, you have 4 total amoebas, which is what we want, and we used 2 instructions. You can check that you can't get 4 amoebas with fewer than 2 instructions, and hence 2 is the minimum, and so the answer is 2.

   Find the minimum number of instructions needed for these instances:

   (a) $K = 23, A = 7, B = 12, C = 16, N = 114$

   (b) $K = 9, A = 7, B = 15, C = 16, N = 76$

   (c) $K = 10, A = 9, B = 12, C = 26, N = 138$

3. There are N people are sitting around a circular table. Let's call them $P_1, P_2, \ldots, P_N$, in order. P_1 is sitting between P_2 and P_N. P_2 is sitting between P_1 and P_3. P_3 is sitting between P_2 and P_4, and so on.

You need to select a non-empty subset of these people, such that no two adjacent people are selected. Find the total number of ways in which you can do so.

For example, suppose $N = 3$, then we have P_1, P_2 and P_3. If we select P_1, then neither P_2 nor P_3 can be selected. So, the only valid selections are $\{P_1\}, \{P_2\}, \{P_3\}$. So there are three possible ways, and hence the answer would be 3.

Suppose $N = 4$, then we have P_1, P_2, P_3 and P_4. The possible subsets are $\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_1, P_3\}, \{P_2, P_4\}$. So the answer would be 6.

Find the answer for the following values of N:

(a) $N = 11$

(b) $N = 13$

(c) $N = 15$

4. Consider an $N \times N$ grid in which very cell has either $+1$ or -1. We call such a grid a Binary grid.

   The Row-Product of any row is defined to be the product of all elements in that single row. So, there are N Row-Products for a given grid. Similarly, the Column-Product of a column is defined to be the product of all elements in that single column. There are N Column-Products in a grid.

   An $N \times N$ Binary grid is called a Magical Grid if exactly one of the N Row-Products is -1, and exactly one of the N Column-Products is -1. That is, the other $N-1$ Row-Products should all be $+1$, and the other $N-1$ Column-Products should all be $+1$.

   Find the number of Magical Grids of size $N \times N$, for the following values of N:

   (a) $N = 3$

   (b) $N = 4$

   (c) $N = 5$