Zonal Informatics Olympiad, 2020

Instructions to candidates

1. The question paper carries 80 marks, broken up into four problems of 20 marks each. Each problem has three Test Cases. If you solve all three Test Cases correctly, you get 20 marks for that problem. Otherwise, you get 5 marks for each Test Case that you solve correctly.

2. All the $4 \times 3 = 12$ Test Cases appear as separate Questions in the right panel ("Question Palette").

   The first three Questions correspond to the three Test Cases of Problem 1, the next three correspond to the three Test Cases of Problem 2 and so on.

   A question icon turning green in the Question Palette, does not mean that it is correct. It just denotes that you have attempted it. All the questions will be evaluated later.

3. Attempt all questions. There are no optional questions.

4. There are no negative marks.

5. All expected answers are integers. Type in only the integer. So, if your answer is "162", enter only "162". Not "0162", or "162.0", etc.

6. Remember to save each answer. Only your final saved answers will be considered.

7. Near the top-right corner, you should be able to see a calculator icon. Clicking it pops up a calculator which you may use.

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Questions

1. The Manhattan Distance between two points (a, b) and (c, d) is given by $|a - c| + |b - d|$, where $|u - v|$ refers to the absolute value of $(u - v)$. For example, the Manhattan Distance between the points $(2, 3)$ and $(-1, 7)$ is $|2 - (-1)| + |3 - 7| = |3| + |-4| = 3 + 4 = 7$.

   Given an integer S, your task is to find the number of points (x, y), where both x and y are integers, such that the Manhattan Distance between (x, y) and $(0, 0)$ is at most S.

   For example, suppose $S = 1$. The only point whose Manhattan Distance from $(0, 0)$ is exactly 0 is $(0, 0)$. The set of points whose Manhattan Distance from $(0, 0)$ is exactly 1 is $\{1, 0), (0, 1), (-1, 0), (0, -1)\}$. Thus, there are 5 points whose Manhattan Distance from $(0, 0)$ is at most 1, and so the answer for $S = 1$ is 5.

   Find the number of points whose Manhattan Distance from $(0, 0)$ is at most S for the following values of S:

   (a) $S = 4$

   (b) $S = 10$

   (c) $S = 23$

2. Suppose you have a rectangular sheet of paper with length L and breadth B. The dimensions are denoted (L, B). Given such a sheet of paper, you can perform two possible folding operations on it:

   1. If L is even, you can fold it length-wise, to transform it into a sheet of paper with length $L/2$ and breadth B, which is $(L/2, B)$.
   2. If B is even, you can fold it breadth-wise to transform it into a sheet of paper with length L and breadth $B/2$, which is $(L, B/2)$.

   For example, starting with a sheet of paper $(12, 36)$, you can go to $(6, 36)$, then $(6, 18)$, then $(6, 9)$, and then $(3, 9)$, after which no operations are possible. In this case, you performed 4 operations. There are other ways you could have decided to fold it. For example, at the beginning, you could have decided to go to $(12, 18)$ instead of $(6, 36)$. But you can check that the maximum number of operations that you can perform starting with $(12, 36)$ is 4.

   You are given an integer S. Your aim is to find two integers (L, B) such that $L + B \leq S$, and the number of folding operations that can be performed starting with (L, B) is maximized. The value you have to report is the maximum number of folding operations possible for a given input S.

   For example, suppose S is 4. Then the possible starting dimensions are:

   - $(1, 1)$ — Starting from this, you cannot perform any operations.
   - $(1, 2)$ — Starting from this, you can perform one operation and go to $(1, 1)$.
   - $(1, 3)$ — Starting from this, you cannot perform any operations.

- $(2, 1)$ — Starting from this, you can perform one operation and go to $(1, 1)$.
- $(2, 2)$ —- Starting from this, you can perform a maximum of two operations and go to $(2, 1)$ and then $(1, 1)$.
- $(3, 1)$ —- Starting from this, you cannot perform any operations.

Among all these possibilities, the maximum number of operations that you can perform is 2, so the answer for $S = 4$ is 2.

Find the maximum number of folding operations possible for the following values of S:

(a) $S = 9$

(b) $S = 58$

(c) $S = 2000$

3. As you know, students from both ZIO and ZCO get selected to write INOI. It is 100 years from now, and INOI-2120 is organized as follows. Each student takes part in exactly one of ZIO or ZCO. We know the scores of all the students in ZIO and ZCO. No two ZIO scores are the same. No two ZCO scores are the same.

   The ZIO and ZCO qualifying scores are calculated as follows: You select a qualifying score X for ZIO and a qualifying score Y for ZCO. All students who score $\geq X$ in ZIO are selected for INOI, and all students who score $\geq Y$ in ZCO get selected for INOI. You want the total number of students selected to be exactly K, and you also want to minimize $|X - Y|$, where $|X - Y|$ refers to the absolute value of $(X - Y)$, so that both exams seem equally difficult. If there are multiple valid pairs (X, Y) which minimize $|X - Y|$, you want the pair which maximizes $(X + Y)$. The final answer is the sum $(X + Y)$ that is maximum among all X, Y with minimum value $|X - Y|$.

   For example, suppose the scores in ZIO are $\{4, 14, 3, 20, 43\}$, the scores in ZCO are $\{6, 2, 50, 27\}$, and $K = 5$. If you choose $X = 10$ and $Y = 23$, you will select 3 students from ZIO $(14, 20, 43)$ and 2 students from ZCO $(50, 27)$, so you would have selected exactly K students. But $|X - Y|$ is $|10 - 23| = 13$. You can do better than this, by selecting $X = 12$ and $Y = 12$. This still selects exactly K students, and now $|X - Y|$ is 0, which is better than 13, but $(X + Y)$ is 24. An even better choice is to select $X = 14$ and $Y = 14$. This still selects exactly K students, and has $|X - Y|$ as 0, but $(X + Y)$ increases to 28. You can check that you cannot do any better, and so the answer for this instance is 28.

   Find the best sum $(X + Y)$ for the following instances:

   (a) Scores in ZIO: $\{29, 60, 5, 31, 23, 22\}$
   Scores in ZCO: $\{18, 1, 22, 9, 2, 8, 35\}$
   $K = 6$

   (b) Scores in ZIO: $\{21, 10, 9, 45, 7, 12, 14, 47, 29, 17\}$
   Scores in ZCO: $\{29, 5, 8, 46, 1, 27, 13, 7, 32, 2, 15, 12\}$
   $K = 11$

(c) Scores in ZIO: $\{47, 28, 49, 35, 52, 38, 43, 39, 34, 57, 20, 18, 48\}$
Scores in ZCO: $\{33, 46, 28, 51, 39, 36, 44, 21, 55, 37, 59, 38, 47, 40\}$
$K = 16$

4. Given an integer N, consider the set $\{\mathit{floor}(N/1),\ \mathit{floor}(N/2),\ \mathit{floor}(N/3),\ \ldots,\ \mathit{floor}(N/N)\}$, where $\mathit{floor}(x)$ is the greatest integer less than or equal to x. You need to find the number of distinct integers in this set.

   For example, suppose N is 5. Then, the set is $\{\mathit{floor}(5/1),\ \mathit{floor}(5/2),\ \mathit{floor}(5/3),\ \mathit{floor}(5/4),\ \mathit{floor}(5/5)\} = \{\mathit{floor}(5),\ \mathit{floor}(2.5),\ \mathit{floor}(1.666\ldots),\ \mathit{floor}(1.25),\ \mathit{floor}(1)\} = \{5, 2, 1, 1, 1\}$. There are 3 distinct elements in this set $(1, 2, 5)$, and so the answer for this would be 3.

   Find the number of distinct integers in the set $\{\mathit{floor}(N/1), \ldots, \mathit{floor}(N/N)\}$ for the following values of N:

   (a) $N = 38$

   (b) $N = 146$

   (c) $N = 2808$