Zonal Informatics Olympiad, 2024

Instructions to candidates

1. The question paper carries 80 marks, broken up into four problems of 20 marks each. Each problem has three Test Cases. If you solve all three Test Cases correctly, you get 20 marks for that problem. Otherwise, you get 5 marks for each Test Case that you solve correctly.
2. All the $4 \times 3 = 12$ Test Cases appear as separate Questions in the right panel ("Question Palette").
   The first three Questions correspond to the three Test Cases of Problem 1, the next three correspond to the three Test Cases of Problem 2 and so on.
   A question icon turning green in the Question Palette, does not mean that it is correct. It just denotes that you have attempted it. All the questions will be evaluated later.
3. Attempt all questions. There are no optional questions.
4. There are no negative marks.
5. All expected answers are integers. Type in only the integer. So, if your answer is "162", enter only "162". Not "0162", or "162.0", etc.
6. Remember to save each answer. Only your final saved answers will be considered.
7. Near the top-right corner, you should be able to see a calculator icon. Clicking it pops up a calculator which you may use.

Problem 1

There is a hidden array $A_1, ...A_N$. There are 2^N subsets of elements in this array. You are given the sum of elements for each of these 2^N subsets in a list S. You have to compute $A_1 + A_2 + \ldots + A_N$.

Compute $A_1 + A_2 + \ldots + A_N$ in each of the following cases.

(a) $N = 3$

$S = [-5, -3, -2, 0, 0, 2, 3, 5]$

(b) $N = 6$

$$S = [\ 0, 0, 1, 1, 2, 2, 3, 3, 3, 3, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 13, 13, 14, 14, 14, 14, 15, 15, 16, 16, 17, 17]$$

(c) $N = 6$

$$S = [\ -19, -18, -16, -15, -14, -13, -12, -11, -11, -11, -10, -10, -10, -9, -9, -8, -8, -7, -7, -7, -6, -6, -6, -5, -5, -4, -4, -4, -3, -3, -3, -3, -2, -2, -2, -2, -1, -1, -1, 0, 0, 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 6, 7, 8, 9, 10, 11, 13, 14]$$

Problem 2

There are N pillars, numbered 1 to N. Each pillar i has height H_i, and is composed of H_i stone slabs stacked on top of each other.

A sequence of pillars is called *beautiful* if the height of each pillar is one more than the height of the previous pillar. For example, $[3, 4, 5, 6, 7]$, $[10, 11, 12, 13]$ and $[4, 5]$ are beautiful sequences, while $[6, 5, 4, 3]$, $[2, 4, 7, 9]$ and $[1, 2, 3, 5, 4]$ are not beautiful.

You can perform the following operations on a sequence:

- Add one stone slab to pillar i. That is, increase H_i by 1.
- Remove one stone slab from pillar i, provided this does not remove all stone slabs from the pillar. That is, reduce H_i by 1 if $H_i > 1$.

For each of the following sequences of pillars, find the minimum number of operations required to make the sequence beautiful.

(a) $N = 3$

$H = [2, 4, 6]$

(b) $N = 12$

$H = [1, 2, 3, 1, 1, 5, 1, 3, 5, 3, 11, 11]$

(c) $N = 20$

$H = [\ 12, 21, 13, 9, 19, 17, 15, 18, 22, 19,$
$17, 19, 15, 20, 24, 17, 35, 25, 25, 29]$

Problem 3

N nodes, numbered 0 to $N-1$, are connected by $N-1$ edges to form a tree. In a tree, you can reach any node j from any node i via a unique sequence of edges.

The edges are denoted by a matrix $P = [p_1, p_2, \ldots, p_{N-1}]$ which says that node p_i is connected to node i. Thus, if $N = 3$ and $P = [2, 0]$, the set of nodes is $[0, 1, 2]$ and P says that node 2 is connected to node 1 and node 0 is connected to node 2.

You are also given a sequence of $N-1$ edge weights W. If we fix an assignment of these $N-1$ edge weights to the $N-1$ edges, we can define the distance $dist(i,j)$ between any pair of nodes i and j as the sum of the edge weights on the unique path from i to j.

Given an assignment of weights to the edges, the *total distance* of the tree is the sum of the pairwise distances between the nodes. Formally, the total distance is

$$\sum_{0 \le i < j \le N-1} dist(i,j)$$

.

Our goal is to compute the maximum total distance among all possible assignments of the given weights to the edges. Compute this quantity in the following cases.

(a) $N = 5$

$P = [0, 0, 0, 1]$

$W = [1, 1, 1, 1]$

(b) $N = 10$

$P = [0, 1, 2, 3, 4, 5, 6, 7, 8]$

$W = [10, 12, 2, 3, 6, 5, 8, 9, 11]$

(c) $N = 15$

$P = [0, 0, 0, 1, 1, 2, 3, 4, 0, 6, 6, 6, 11, 11]$

$W = [9, 8, 19, 5, 6, 2, 2, 4, 5, 20, 25, 11, 15, 13]$

Problem 4

There are N balls, $B_1, B_2, \ldots, B_N$. Each ball B_i has a colour C_i. The colours are not necessarily distinct. For instance, we could have 3 balls with colours $[red, blue, red]$.

We can form 2^N subsets of balls. A subset is considered *good* if the balls in the subset can be arranged in a line in at least one way such that no pair of adjacent balls are of the same colour.

In each of the following situations, compute the number of good subsets. The balls are listed in the order $[B_1, B_2, \ldots, B_N]$ in terms of their colour.

Note: Any subset of size 0 and 1 is considered good.

(a) $N = 4$

$C = [red, red, green, green]$

(b) $N = 8$

$C = [red, red, red, green, green, green, blue, blue]$

(c) $N = 20$

$$\begin{aligned} C = [\ & red, green, green, green, blue, blue, blue, \\ & yellow, yellow, yellow, yellow, yellow, yellow, \\ & purple, purple, purple, purple, \\ & purple, purple, purple] \end{aligned}$$