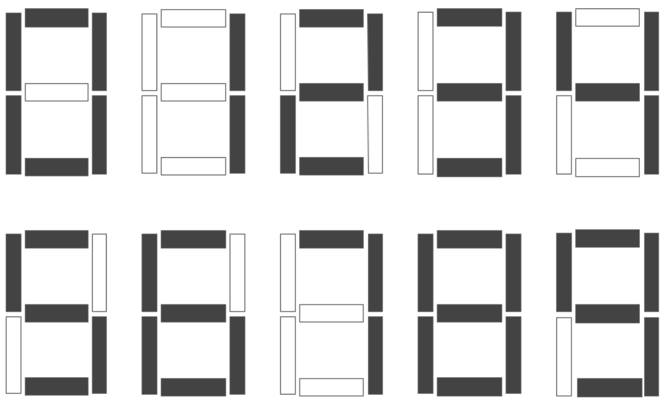
Zonal Informatics Olympiad, 2025

Instructions to candidates

1. The question paper carries 80 marks, broken up into four problems of 20 marks each. Each problem has three Test Cases. If you solve all three Test Cases correctly, you get 20 marks for that problem. Otherwise, you get 5 marks for each Test Case that you solve correctly.
2. All the $4 \times 3 = 12$ Test Cases appear as separate Questions in the right panel ("Question Palette").
   The first three Questions correspond to the three Test Cases of Problem 1, the next three correspond to the three Test Cases of Problem 2 and so on.
   A question icon turning green in the Question Palette, does not mean that it is correct. It just denotes that you have attempted it. All the questions will be evaluated later.
3. Attempt all questions. There are no optional questions.
4. There are no negative marks.
5. All expected answers are integers. Type in only the integer. So, if your answer is "162", enter only "162". Not "0162", or "162.0", etc.
6. Remember to save each answer. Only your final saved answers will be considered.
7. Near the top-right corner, you should be able to see a calculator icon. Clicking it pops up a calculator which you may use.

Problem 1

There are N displays placed in a row. Each display can show a single digit from 0 to 9. Thus, together, N displays can show any non-negative number that is less than 10^N. Each display has 7 lights. The figure below shows how each digit is represented and how many lights there are per digit.



Digit	0	1	2	3	4	5	6	7	8	9
No. of Lights	6	2	5	5	4	5	6	3	7	6

Figure 1: Representation of digits on a display, and number of lights required to display each digit

An *operation* consists of turning on or turning off a single light. You can convert any digit to any another one using a sequence of operations. For example, a 2 can be converted to a 6 with 3 operations (two lights being turned on and one being turned off) as shown in the next figure.

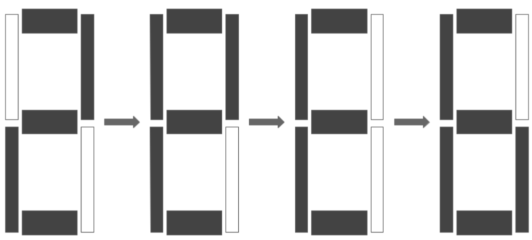


Figure 2: Operations required to make 6 from 2

Given the number of displays N and a number X shown on them, determine the minimum number of operations you will have to perform to obtain a number greater than X. Note that N is constant for a given test case, and if a number is smaller than 10^{N-1}, it will be padded with leading 0s.

(a) $N = 2$, $X = 77$

(b) $N = 10$, $X = 0000942274$

(c) $N = 15$, $X = 127932412749752$

Problem 2

You need to select a passcode consisting of N numeric digits. Each digit can be between 0 and 9, both inclusive, and your passcode may have leading 0s. Therefore, there are 10^N possible passcodes. Let D_i denote the i^{th} digit of your passcode.

To make your passcode difficult to guess, you make sure that:

- No two adjacent digits are equal. That is, for all $1 \le i < N$, $D_i \neq D_{i+1}$.
- For any three consecutive digits, they are not all increasing. That is, there must be no $1 \le i \le N-2$ such that $D_i < D_{i+1}$ and $D_{i+1} < D_{i+2}$.
- For any three consecutive digits, they are not all decreasing. That is, there must be no $1 \le i \le N-2$ such that $D_i > D_{i+1}$ and $D_{i+1} > D_{i+2}$.

For a given length N, what is the number of passcodes you can create of that length? **Find this value modulo** 997**.**

(a) $N = 3$

(b) $N = 6$

(c) $N = 11$

Problem 3

There are N towers in a row. Tower i contains H_i blocks.

You want to make the row of towers special. A row of towers is special if there exist no three integers $1 \leq i < j < k \leq N$ such that $H_i > H_j$ and $H_j < H_k$.

To do this, you can perform some operations. In each operation, you can pick some index i such that $1 \leq i \leq N$ such that $H_i \geq 1$ and then decrease H_i by one, that is set $H_i = H_i - 1$.

What is the minimum number of operations you need to perform? **If it is impossible to make the row of towers special using any number of operations, you should answer -1 (minus 1).**

(a) $N = 5$, $H = [3, 1, 3, 4, 1]$

(b) $N = 12$, $H = [12, 9, 10, 7, 11, 9, 4, 6, 15, 5, 9, 12]$

(c) $N = 25$, $H = [22, 26, 7, 25, 26, 14, 22, 26, 8, 20, 2, 22, 24, 6, 6, 10, 11, 22, 20, 22, 3, 10, 28, 25, 15]$

Problem 4

A distant country has N cities numbered $\{1, 2, \ldots, N\}$. These cities are connected by $N - 1$ roads such that it is possible to travel between any pair of cities using these roads.

For some non-negative integer K, you want to create an array of K pairs of cities satisfying the following constraints. Let the i^{th} pair in the array be (A_i, B_i).

- $1 \leq A_i < B_i \leq N$ for all $1 \leq i \leq K$
- For all $1 \leq i < K$, either:
  - $A_i < A_{i+1}$, or
  - $A_i = A_{i+1}$ and $B_i < B_{i+1}$.
- For each $1 \leq i \leq K$, consider any road in the unique simple path between A_i and B_i. For each of the $N - 1$ roads, this road must occur **exactly** once over all K simple paths.

How many different arrays of pairs can you create? Two arrays of pairs are considered different if there are a different number of pairs in the array, or if there is some index i such that the i^{th} pair in the first array is not equal to the i^{th} pair in the second array. Two pairs are considered equal if their first elements are equal to each other and their second elements are equal to each other. **Find the number of different arrays of pairs modulo 997.**

Consider for example $N = 4$ with the following 3 roads:

- A road connecting city 1 and city 2
- A road connecting city 1 and city 3
- A road connecting city 3 and city 4

Then, the possible arrays are:

- $[(2, 4)]$ (here, $K = 1$)
- $[(1, 2), (1, 4)]$ (here, $K = 2$)
- $[(2, 3), (3, 4)]$ (here, $K = 2$)
- $[(1, 2), (1, 3), (3, 4)]$ (here, $K = 3$)

Therefore, there are four possible arrays of pairs, and the answer for this testcase would be 4.

The input consists of the integer N, the number of cities, followed by an array of integers P of length N, such that $P[1] = -1$ and $1 \leq P[i] < i$ for all $2 \leq i \leq N$. For each i such that $2 \leq i \leq N$, there is a road connecting cities $P[i]$ and i.

Please remember to output your answer modulo 997.

(a) $N = 30$, $P = [-1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]$

(b) $N = 15$, $P = [-1, 1, 1, 2, 3, 1, 2, 2, 3, 3, 5, 5, 4, 11, 12]$

(c) $N = 30$, $P = [-1, 1, 1, 2, 2, 3, 1, 1, 2, 2, 2, 3, 4, 1, 1, 1, 2, 3, 3, 1, 4, 4, 5, 6, 5, 6, 6, 3, 3, 10]$